*In*Validity of Relative Density for Quality Control of Cohesionless Soils

Babak Hamidi
Serge Varaksin
Hamid Nikraz

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Outline

- History & development of Relative Density
- How reliable is the concept?
- How reliable are the correlations?
  - Standard Penetration Test
  - Cone Penetration Test

History

- 1925: Realization of concept
  - TERZAGHI, K. (1925) *Erdbaumechanik auf Bodenphysikalisher Grundlage,* Vienna, Deuticke
- 1948: Aim
  - to bring the behaviour characteristics of soils together on a common basis in consistent and practically useful relations and to provide a tool for communications between engineers
  - BURMISTER, D. M. (1948) The Importance and Practical Uses of Relative Density in Soil Mechanics. *American Society for Testing and Materials,* 48, Philadelphia, PA, USA, 1249-1268.

History

- An appropriate means to define the looseness and denseness of sand or sand-gravel soils in a meaningful way because important properties were assumed to correlate quite well by this means

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History

- 1954: Formation of ASTM Committee D-18, Subcommittee 3, Section D for determining the minimum and maximum densities of sand and gravel soils
- 1964: Approval of standard by D-18
- 1969: ASTM standard for Relative Density
  - ASTM (1969) Test Method for Relative Density of Cohesionless Soils. *D2049-69.*
- 1984: Withdrawal of ASTM D2049-69

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History

- 2011
  - ASTM D4253-00 (Reapproved 2006) Standard Test Methods for Maximum *Index* Density and Unit Weight of Soils Using a Vibratory Table.
  - ASTM D4254-00 (Reapproved 2006) Standard Test Methods for Minimum *Index* Density and Unit Weight of Soils and Calculation of Relative Density.

Definition: includes three parameters

$$D_d = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

e_{max}= maximum index void ratio or the reference void ratio of a soil at the minimum index density/unit weight.

e_{min}= minimum index void ratio or the reference void ratio of a soil at the maximum index density/unit weight.

e= the in situ or stated void ratio of a soil deposit or fill.

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Alternative definitions

$$D_d = \frac{\rho_{dmax}(\rho_d - \rho_{dmin})}{\rho_d(\rho_{dmax} - \rho_{dmin})} \times 100$$

$$D_d = \frac{\gamma_{dmax}(\gamma_d - \gamma_{dmin})}{\gamma_d(\gamma_{dmax} - \gamma_{dmin})} \times 100$$

ρ_d or Υ_d = dry density/unit weight of a soil deposit or fill at the given void ratio.

ASTM Limits of Application

- Soil can contain up to *15%*, by dry mass, of soil particles passing a 75-µm sieve, provided they still have *cohesionless*, *free-draining* characteristics.
- For determination of ρ_{dmin}, Υ_{dmin}, ρ_d or Υ_d
  - 3accepted methods are applicable to soil in which 100% of soil particles pass respectively a 75, 19 & 9.5 mm sieves.

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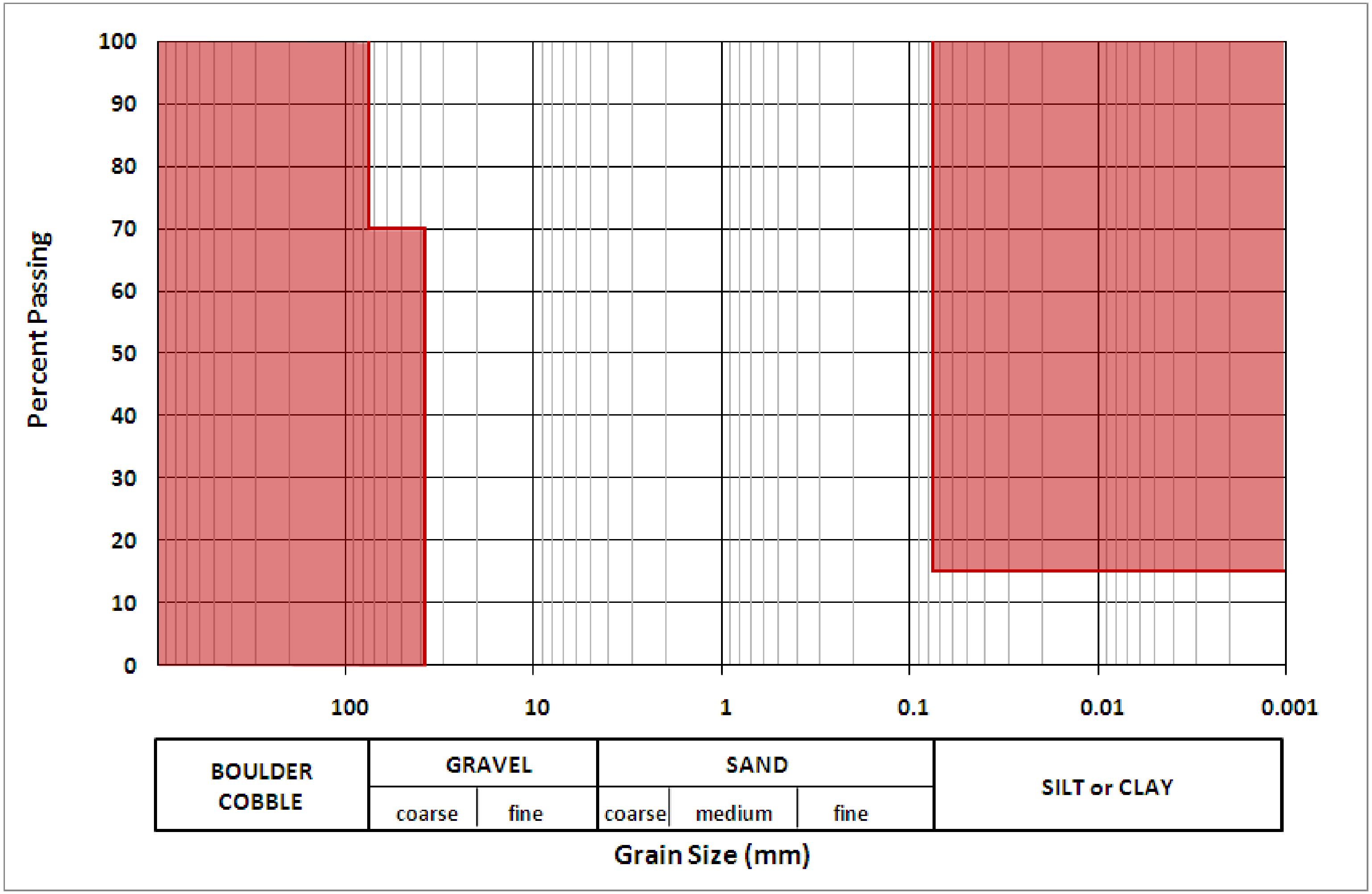
ASTM Limits of Application



ASTM words of caution

- For many types of *free-draining, cohesionless* soils, these test methods cause a moderate amount of degradation (particle breakdown) of the soil. When degradation occurs, typically there is an increase in the maximum index density/ unit weight obtained, and comparable test results may not be obtained when different size molds are used to test a given soil.

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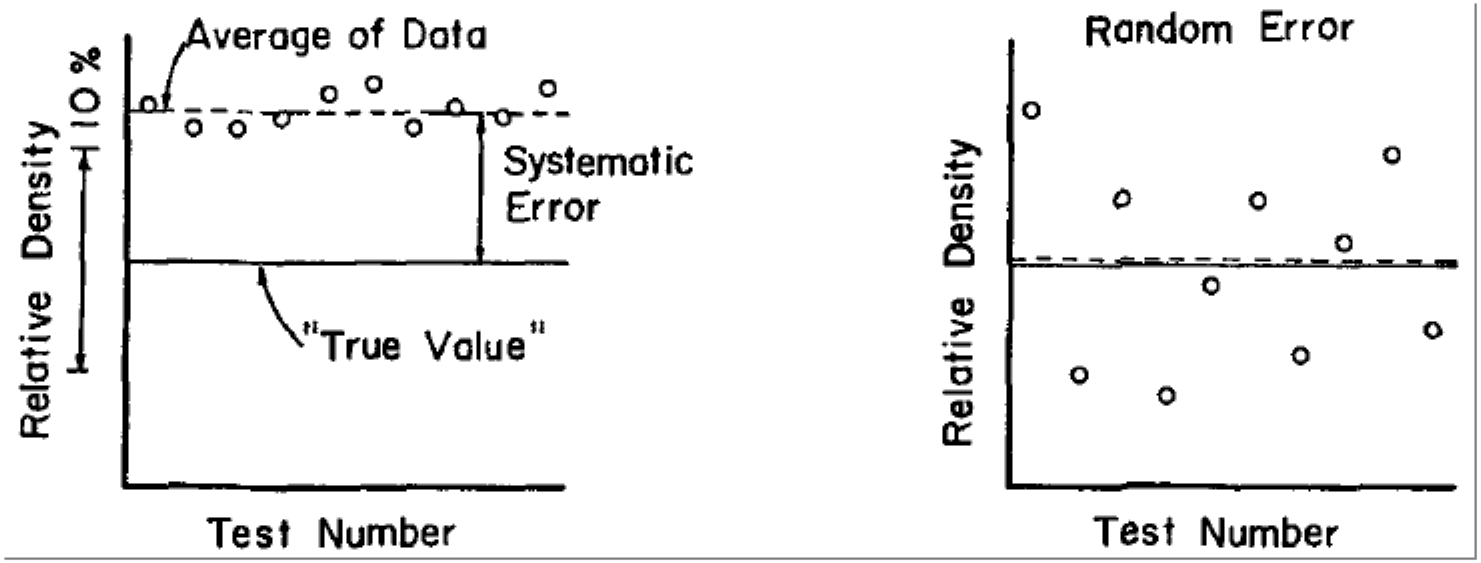
ASTM words of caution

- The engineering properties, such as strength, compressibility, and permeability of a given soil, compacted by various methods to a given state of compactness can vary considerably. Therefore, considerable engineering judgment must be used in relating the engineering properties of soil to the state of compactness.
  - Note: In addition, there are published data to indicate that these test methods have a high degree of variability.

Errors

- **Systematic error** is a measure of accuracy and the difference between correct value and the measured average of a set of repeated tests.
- **Random error** is the precision of a quantity and is measured by the scatter in the results of a group of repeated tests
- **Mistake**
  - SELIG, E. T. & LADD, R. S. (1973) Evaluation of Relative Density Measurements and Applications. *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils: ASTM STP523-EB.7744-1.*, Los Angeles, 25-30 June 1972, 487-504.

Errors



SELIG, E. T. & LADD, R. S. (1973)

Yoshimi & Tohno (1973)

– YOSHIMI, Y. & TOHNO, I. (1973) Statistical Significance of the Relative Density. *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils: ASTM STP523-EB.7744-1*, Los Angeles, 25-30 June 1972, 74-84.

– D_d is proportional to $(\Upsilon_d - \Upsilon_{dmin})/\Upsilon_d$ →

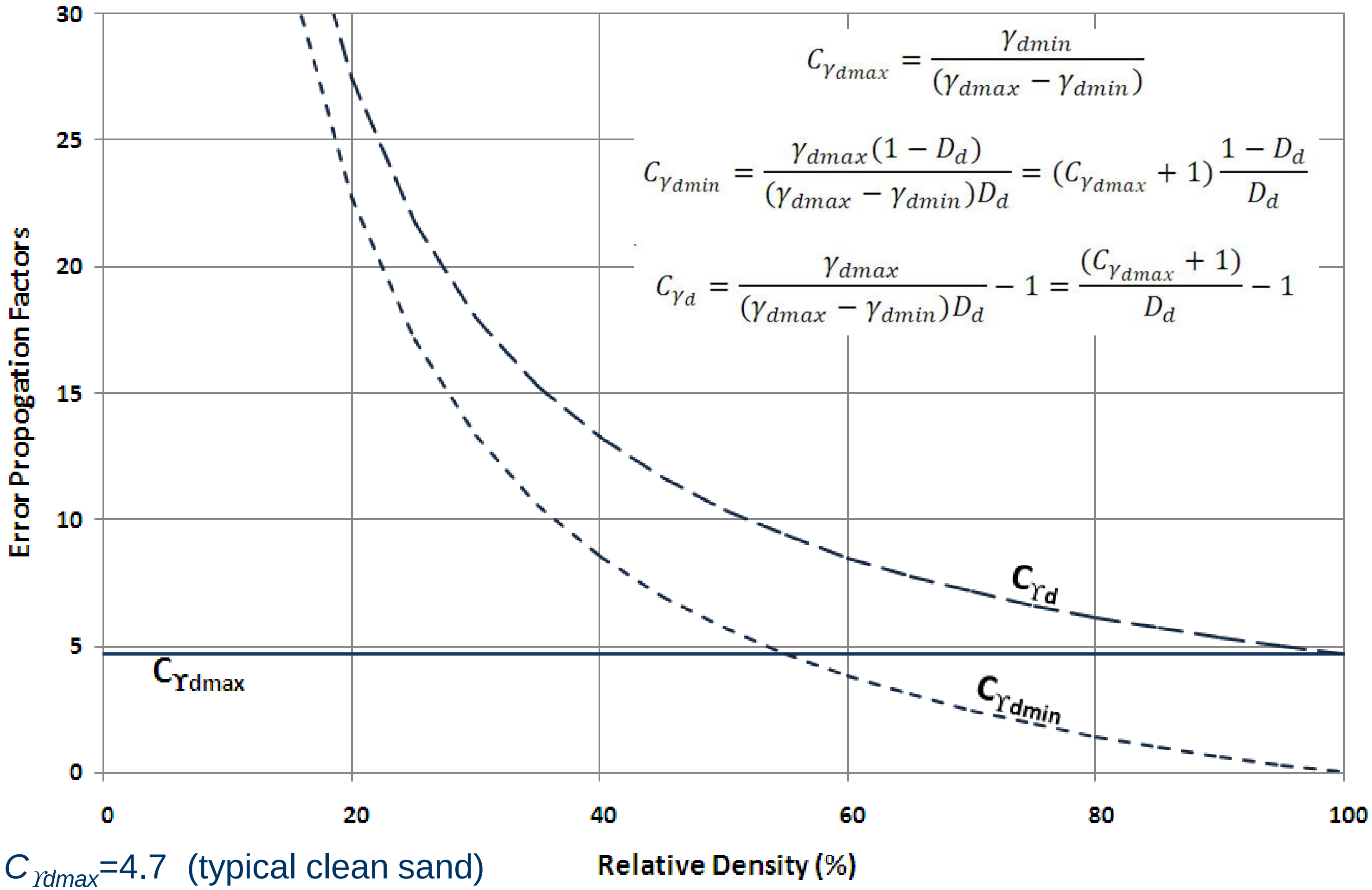
- even a small variation in Υ_d or Υ_{dmin} → considerable variation in D_d when $\Upsilon_d - \Upsilon_{dmin}$ is small; i.e. when D_d is low.
- E.g.: Υ_{dmin}= 13.5 kN/m^3, Υ_{dmax}= 16.37 kN/m^3, & Υ_d= 14.25 kN/m^3, D_d = 30%. If Υ_{dmin} is increased by 1% to 13.635 kN/m^3, D_d reduces to 25.8% which is 14% less than the initial value. In other words, the relative deviation in relative density is 14 times that of Υ_{dmin}.

Yoshimi & Tohno: random errors

$$\left(\frac{S_{D_d}}{D_d}\right)^2 = C_{\gamma_{dmax}}{}^2\left(\frac{S_{\gamma_{dmax}}}{\gamma_{dmax}}\right)^2 + C_{\gamma_{dmin}}{}^2\left(\frac{S_{\gamma_{dmin}}}{\gamma_{dmin}}\right)^2$$

- The terms in the parentheses are coefficients of variation. S_{Dd}, $S_{\gamma dmax}$, $S_{\gamma dmin}$ and $S_{\gamma d}$ are respectively the standard deviations for D_d, γ_{dmax}, γ_{dmin} and γ_d, and $C_{\gamma dmax}$, $C_{\gamma dmin}$ and $C_{\gamma d}$ are error propagation factors.
- Random errors can be reduced to any desired degree by repeating the test and averaging the results.

Yoshimi & Tohno: random errors



$C_{\gamma dmax}$=4.7 (typical clean sand)

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Yoshimi & Tohno: systematic errors

$$\left.\frac{\Delta D_d}{D_d}\right]_{\Delta\gamma dmin=\Delta\gamma=0} = \frac{\gamma_{dmin}}{\gamma_{dmax}-\gamma_{dmin}+\Delta\gamma_{dmax}}\frac{\Delta\gamma_{dmax}}{\gamma_{dmax}}$$

$$\left.\frac{\Delta D_d}{D_d}\right]_{\Delta\gamma dmax=\Delta\gamma=0} = \frac{(1-D_d)\gamma_{dmax}}{D_d(\gamma_{dmax}-\gamma_{dmin}-\Delta\gamma_{dmin})}\frac{\Delta\gamma_{dmin}}{\gamma_{dmin}}$$

$$\left.\frac{\Delta D_d}{D_d}\right]_{\Delta\gamma dmax=\Delta\gamma dmin=0} = \frac{\gamma_d\gamma_{dmin}}{(\gamma_d-\gamma_{dmin})+(\gamma_d+\Delta\gamma_d)}\frac{\Delta\gamma_d}{\gamma_d}$$

$\Delta D_d/D_d$ = relative deviation

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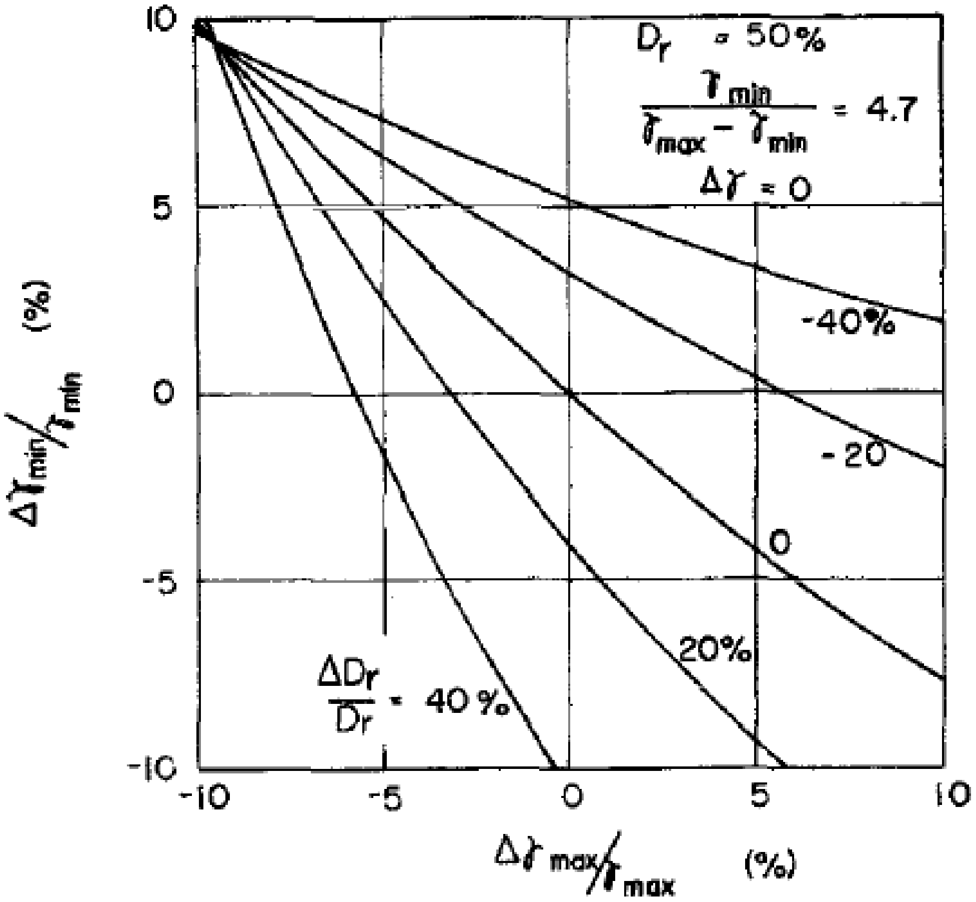
Yoshimi & Tohno: systematic errors

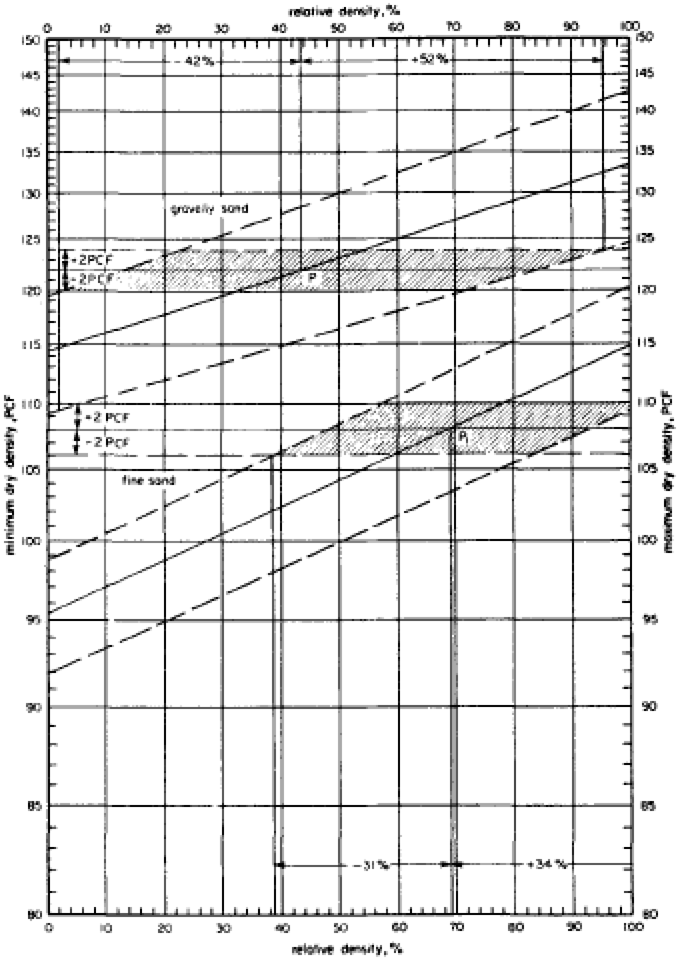


Influence of systematic errors in limiting densities on relative density

Tavenas et al. (1973)

Due to the very large variability of the relative density between laboratories, the comparison of relative densities measured by different laboratories were totally non-significant. There were important practical implications of this fact: all established correlations between relative density and various properties of cohesionless soils such as standard penetration index, point resistance in a static penetration test, friction angle, modulus of compressibility, shear wave velocity, etc., are useless to anyone but the operator who has established them, since that person is the only one who can reproduce the relative density of the considered soil with sufficient accuracy.

Tavenas et al. (1973)



It appeared that due not so much to the variability of the minimum and maximum unit weights but essentially to the formulation of relative density itself, the resulting accuracy of this parameter was so poor that its use was related to major uncertainties (the best case was of ideal material such as the tested fine sand, and was deemed to be practically meaningless in most of the other cases.

TAVENAS, F. A., LADD, R. S. & LA ROCHELLE, P. (1973) Accuracy of Relative Density Measurements: Results of a Comparative Test Program. *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils: ASTM STP523-EB.7744-1*, Los Angeles, 25-30 June 1972, 18-60.

Practical problems of relative density

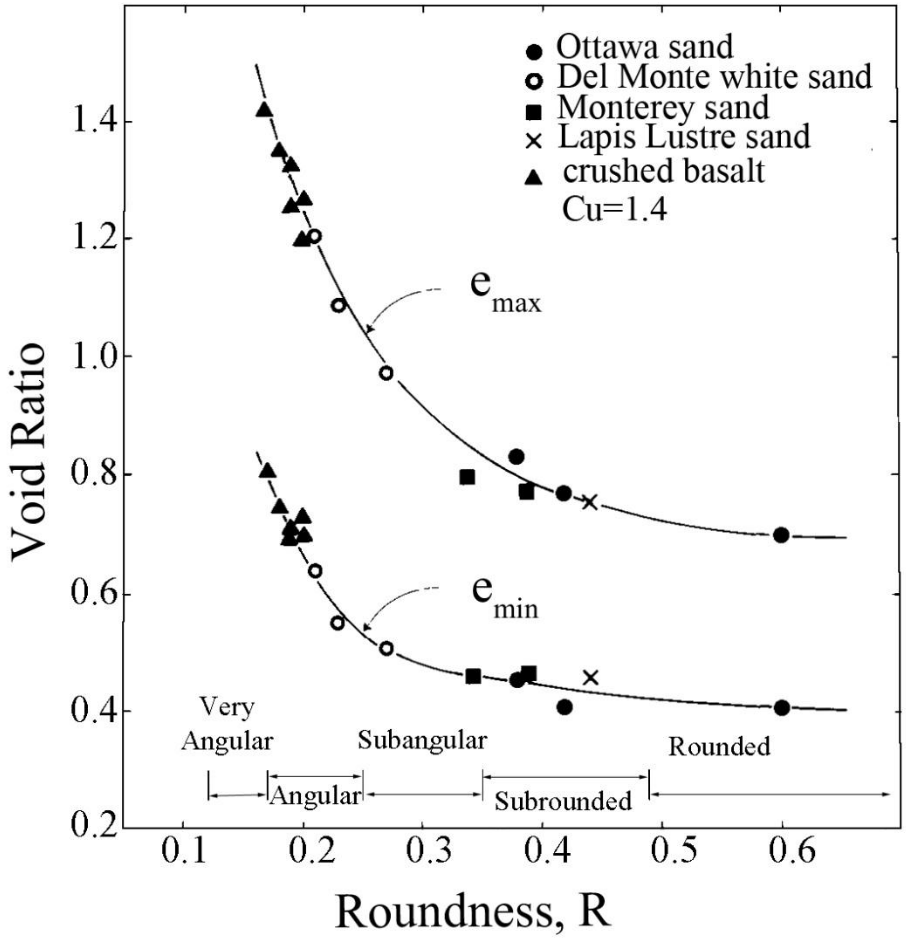
- Difficult & costly to implement at depth
- Difficult & costly to implement below groundwater level
- Costly to repeat sufficient number of times to reduce random errors

→ **Correlations:** a statistical relation between two or more variables such that systematic changes in the value of one variable are accompanied by systematic changes in the other. Correlations are not physical laws or theorems, they are simply statistical relations and only meaningful once their *scatters*, *deviations* and *variances* are known.

Soil properties are not based only on D_d

- Fines content
- Grain size & shape
- Grading & grading curve shape
- Effective vertical or horizontal stresses
- Mineralogy
- Compressibility & crushability
- Cementation
- Over consolidation
- Age

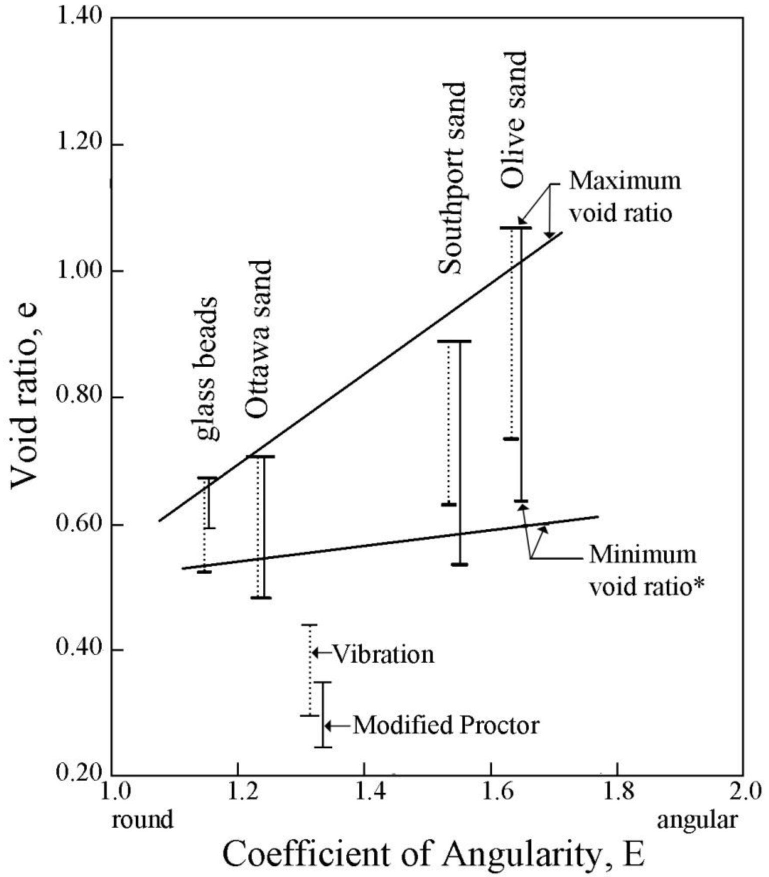
Grain shape



YOUD, T. L. (1973) Factors Controlling Maximum and Minimum Densities of Sands. *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils: ASTM STP523-EB.7744-1*, Los Angeles, 25-30 June 1972, 98-112.

e_{max} & e_{min} as a function of grain with C_u=1.4

Grain shape

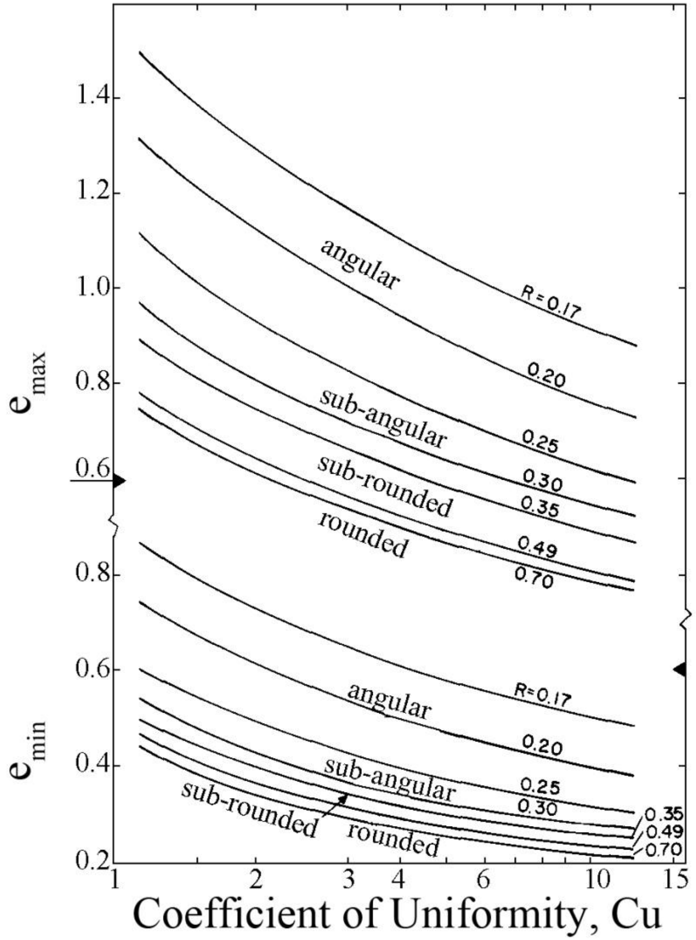


* Minumum void ratio based on modified Proctor Compaction test, except for glass beads

HOLUBEC, I. & D'APPOLONIA, E. (1973) Effect of Particle Shape on the Engineering Properties of Granular Soils. *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils: ASTM STP523-EB.7744-1*, Los Angeles, 25-30 June 1972, 304-318.

Effect of particle shape on e_{max} and e_{min} from gradational and particle shape characteristics

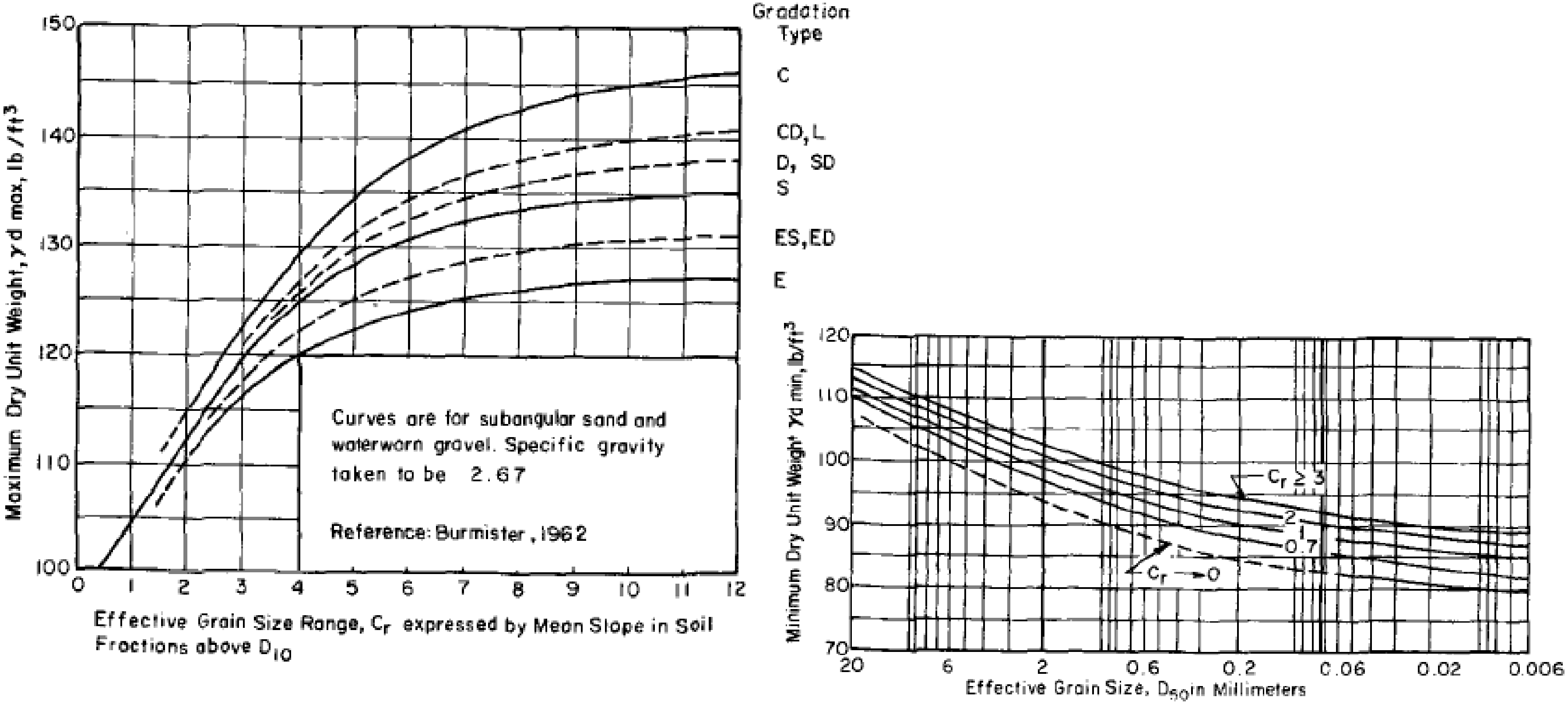
Grain shape & gradation



YOUD, T. L. (1973) Factors Controlling Maximum and Minimum Densities of Sands. *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils: ASTM STP523-EB.7744-1*, Los Angeles, 25-30 June 1972, 98-112.

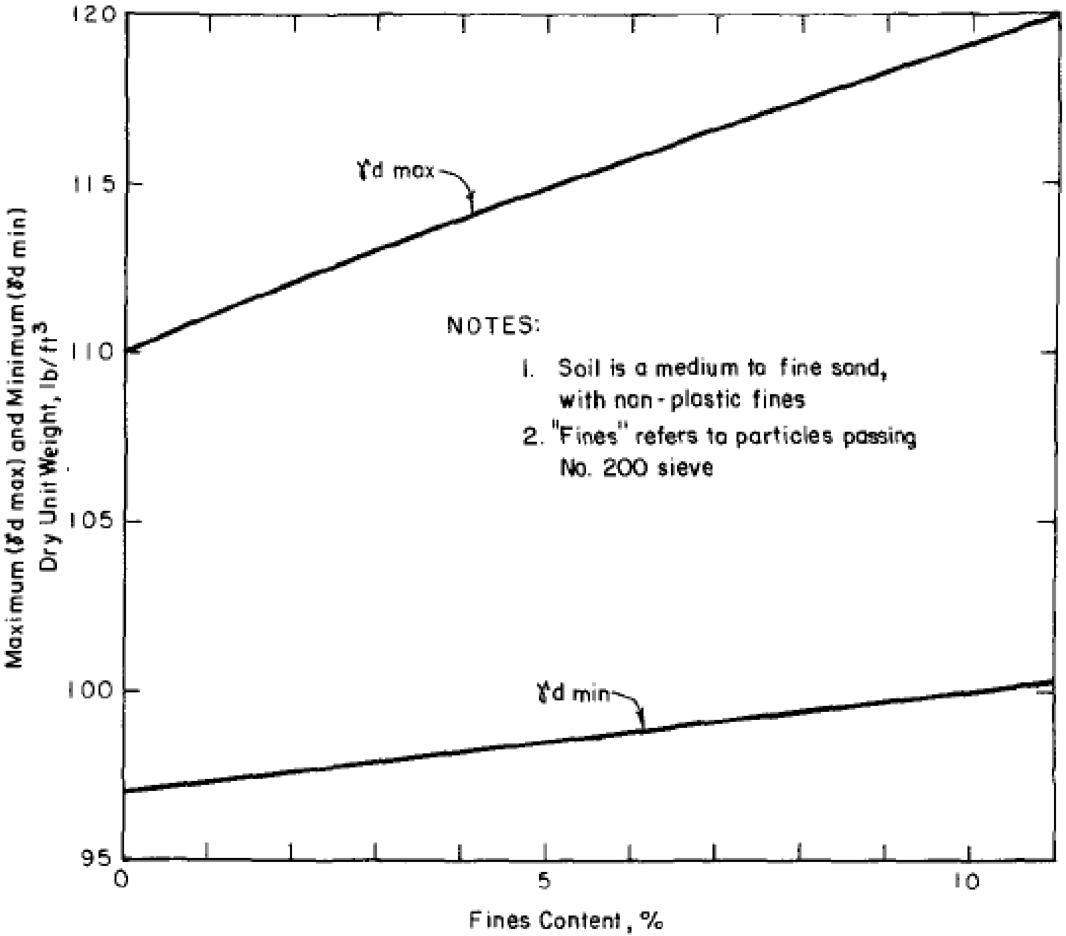
e_{max} and e_{min} from gradational and particle shape characteristics

Particle size



LACROIX, Y. & HORN, H. M. (1973) Direct Determination and Indirect Evaluation of Relative Density and its Use on Earthwork Construction Projects. *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils: ASTM STP523-EB.7744-1, Los Angeles, 25-30 June 1972, 251-280*.

Fines content



LACROIX, Y. & HORN, H. M. (1973) Direct Determination and Indirect Evaluation of Relative Density and its Use on Earthwork Construction Projects. *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils: ASTM STP523-EB.7744-1, Los Angeles, 25-30 June 1972, 251-280.*

Correlation between limiting dry unit weights and fines content

SPT & relative density

How reliable are samples extracted from the SPT split spoon?

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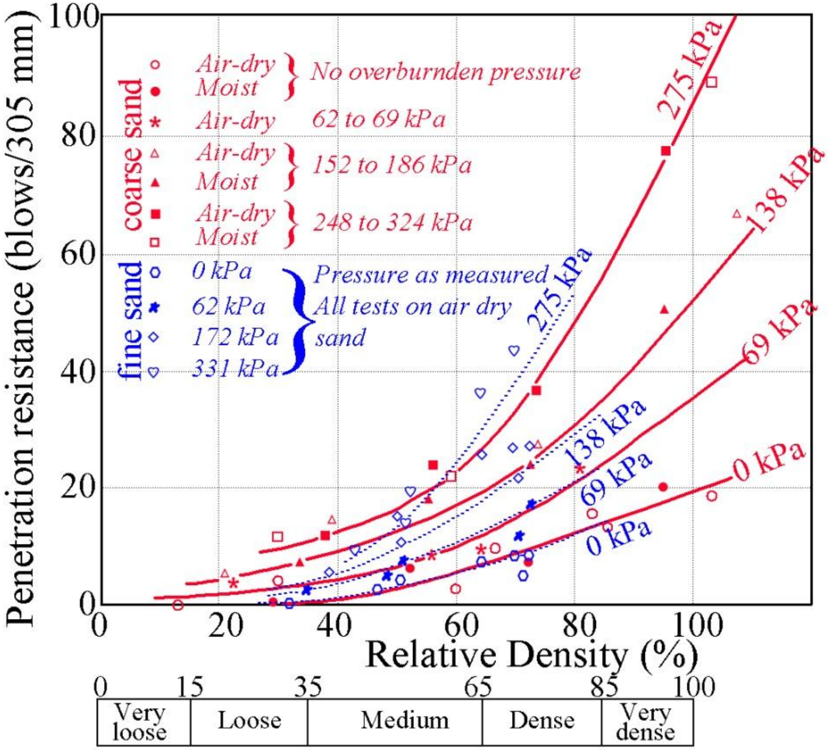
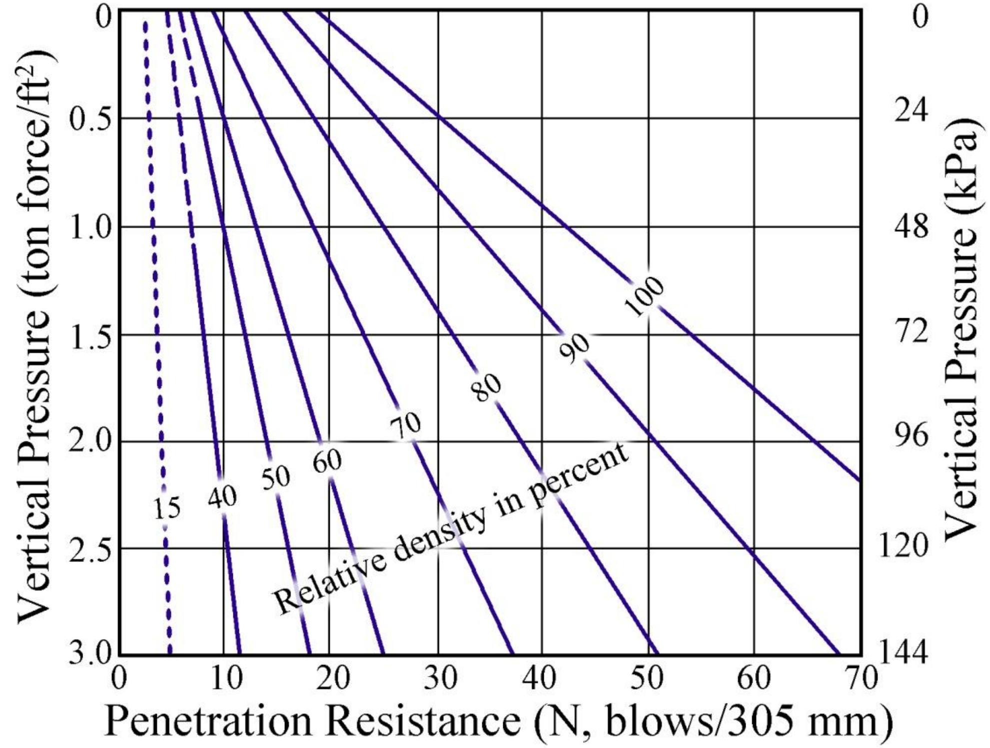
Gibbs & Holtz - Meyerhof (1957) & USBR





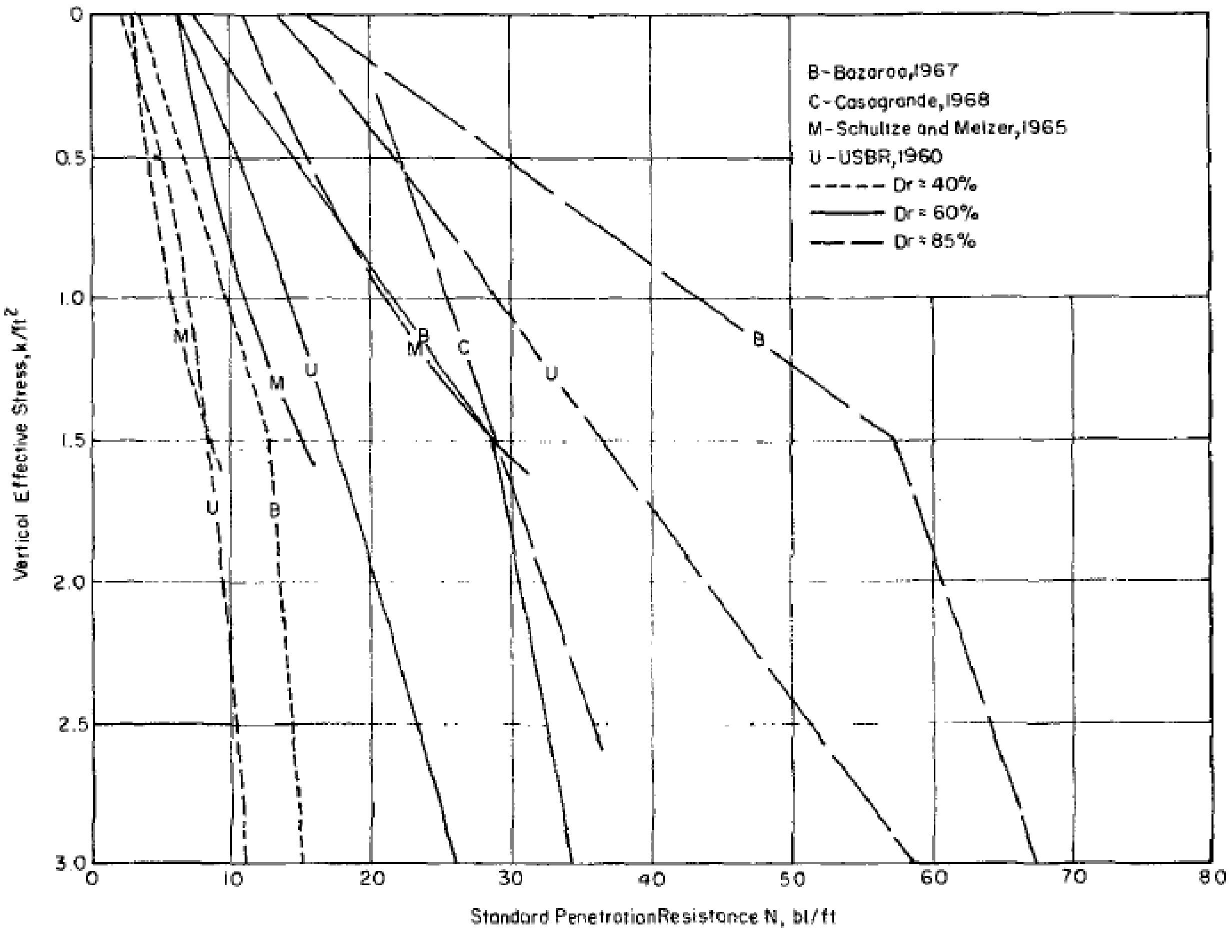
GIBBS, K. J. & HOLTZ, W. G. (1957) Research on Determining the Density of Sands by Spoon Penetration Testing. *4th International Conference on Soil Mechanics and Foundation Engineering,* 1, London, 35-39.

BUREAU OF RECLAMATION (1960) *Earth Manual*, Denver, US Department of the Interior.

$$D_d = \sqrt{\frac{N}{17 + 0.25\sigma'_v}}$$

MEYERHOF, G. G. (1957) Discussion on Research on Determining the Density of Sands by Spoon Penetration Testing. *4th International Conference on Soil Mechanics and Foundation Engineering*, 1, London, 110.

Different correlations yield very different results



Lacroix & Horn (1973)

How good is the SPT for determining D_d?

Holtz (1973): “First, I think that everyone should recognize that the Standard Spoon Penetration Test is a relatively crude test and no one should expect to determine the relative density of sands to the nearest one percent or anything like that. When Mr. Gibbs and I developed a set of correlations to take into account the effect of overburden pressures, we never indicated that the sets of curves developed at that time were necessarily applicable to all cohesionless soils under all conditions. Second, we always stressed the relative density trends indicated by the Standard Penetration Test (SPT) values rather than the specific individual values. Third, I wanted to point out that Mr. Gibbs and I are not particular "promoters" of the SPT, although we think it is useful for certain types of foundation investigations, and at certain stages of investigation.”

HOLTZ, R. D. (1973) Discussion on Determination of Relative Density of Sand Below Groundwater Table. *Evaluation of Relative Density and its Role in Geotechnical Projects Involving Cohesionless Soils: ASTM STP523-EB.7744-1*, Los Angeles, 25-30 June 1972, 376-377.

Skempton (1986)

$$D_d = \sqrt{\frac{N}{a + b\sigma'_v}}$$

Sand	Tested	D_{50}: mm	UC*	Fines: %	D_r:	N_1	$\frac{N_1}{D_r^2}$	$\frac{N}{D_r^2}$	$\frac{ER_r}{60}$	$(N_1)_{60}$	$\frac{(N_1)_{60}}{D_r^2}$	$\frac{N_{60}}{D_r^2}$
PR	Wet	2·0	5·3	0	0·4 0·6 0·8	7·5 19 37	47 53 58	$30 + 22\sigma_v'$	1·1†	8 21 41	52 58 64	$33 + 24\sigma_v'$
GHC	Dry and moist	1·5	5·5	0	0·4 0·6 0·8	6·5 14·5 25	40 40 39	$18 + 22\sigma_v'$				
SCS	Wet	0·51	2·5	4	0·4 0·6 0·8	7 16 29	44 44 45	$21 + 24\sigma_v'$	1·1†	7·5 18 32	48 48 49	$23 + 26\sigma_v'$
RBM	Wet	0·23	1·8	2	0·4 0·6 0·8	5·5 12 21	34 33 33	$16 + 17\sigma_v'$	1·1†	6 13 23	37 36 36	$17 + 19\sigma_v'$
GHF	Dry	0·3	7	14	0·4 0·6 0·8	4·5 12 23	28 33 36	$15 + 18\sigma_v'$				

* Uniformity coefficient.
† Includes a correction for no liners.

SKEMPTON, A. W. (1986) Standard Penetration Test Procedure and the Effects in Sands of Overburden Pressure, Relative Density, Particle Size, Ageing and Overconsolidation. *Geotechnique, 36, 3, 425-447.*

Grain size, age & consolidation

- At a given relative density and overburden pressure, *N* values are higher for sands with larger grain sizes (D_{50})
- Ageing of sand will increase the SPT blow counts
- Over Consolidation

$$D_d = \sqrt{\frac{N}{a + C_{oc} b \sigma'_v}}$$

$$C_{oc} = \frac{1 + 2K_O}{1 + 2K_{ONC}} \qquad K_{ONC} = 1 - \sin\phi' \qquad K_O = K_{ONC}(OCR)^{\sin\phi'}$$

SKEMPTON, A. W. (1986) Standard Penetration Test Procedure and the Effects in Sands of Overburden Pressure, Relative Density, Particle Size, Ageing and Overconsolidation. *Geotechnique, 36, 3, 425-447.*

Tokimatsu & Yoshimi (1983): fines content

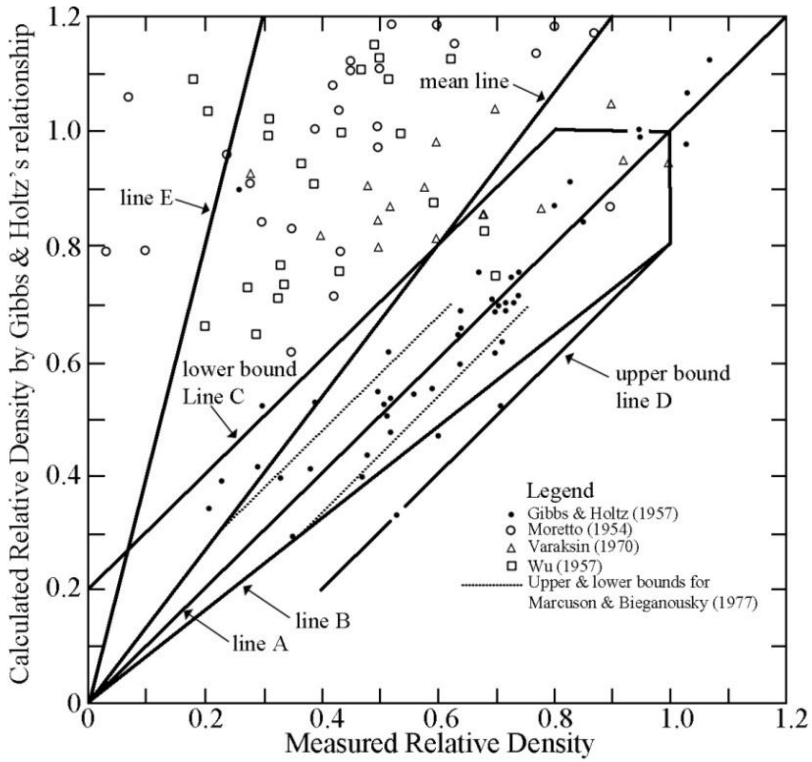
$$D_d = 0.21\sqrt{\frac{N}{0.7 + \frac{\sigma'_v}{98}} + \frac{\Delta N_f}{1.7}}$$

Fines content (%)	ΔN_f
0-5	0
5-10	interpolate
10-	$0.1F_c$+4

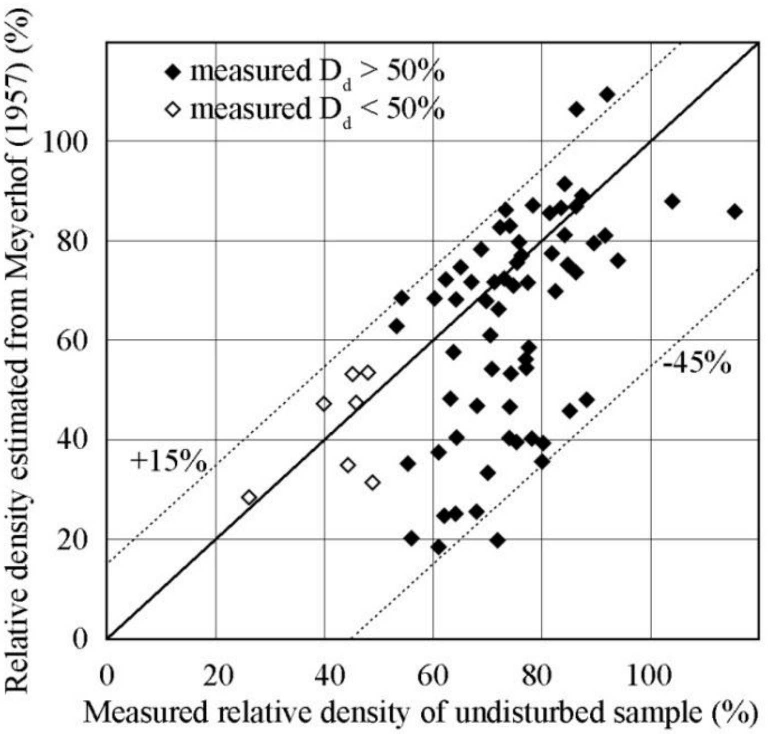
- Tokimatsu and Yoshimi themselves have not demonstrated confidence in their proposed equation and note that its application is yet to be proven

TOKIMATSU, K. & YOSHIMI, Y. (1983) Empirical Correlation of Soil Liquefaction Based on SPT-N Value and Fines Content. *Soils and Foundations, Japanese Society of Soil Mechanics and Foundation Engineering, 23, 4, 56-74.*

Scatter of data

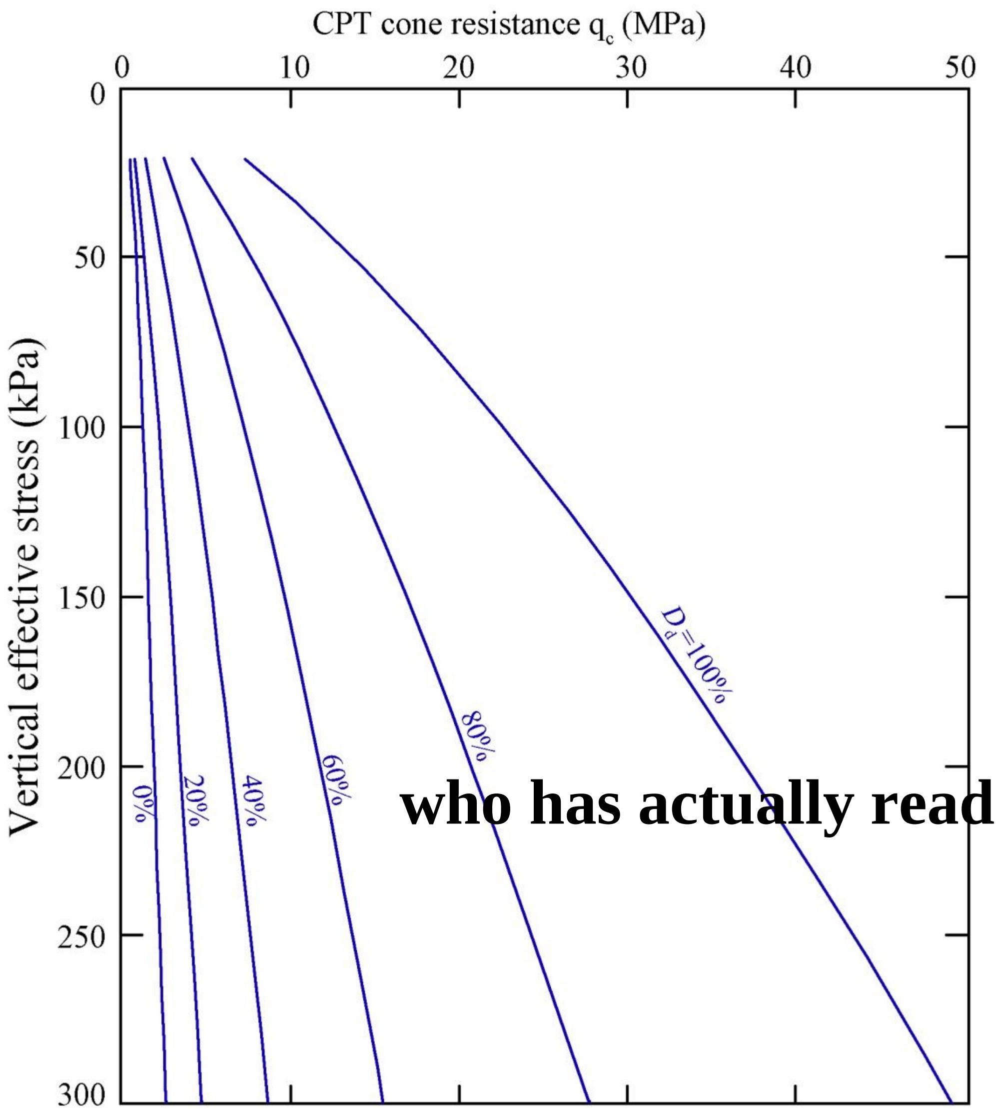


HALDAR, A. & TANG, W. H. (1979) Uncertainty Analysis of Relative Density. *Journal of Geotechnical Engineering, ASCE*, 107, 7 (July), 899-904.



HATANAKA, M. & FENG, L. (2006) Estimating Relative Density of Sandy Soils. *Soils and Foundations, Japanese Society of Soil Mechanics and Foundation Engineering*, 46, 3, 299-313.

What is Schmertmann (1976)?

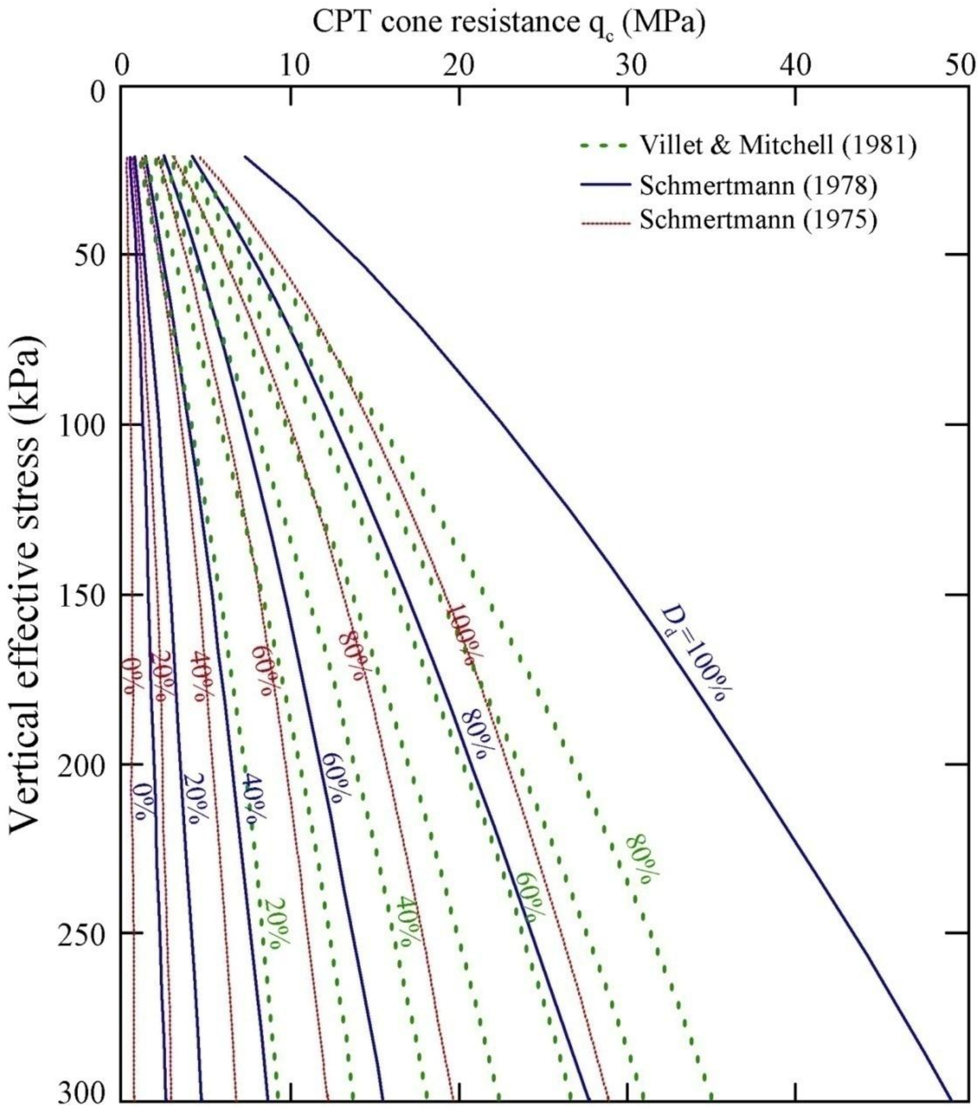


SCHMERTMANN, J. H. (1976) An Updated Correlation Between Relative Density and Fugro-type Electric Cone Bearing qc. Contract report, DACW 38-76-M 6646. Vicksburg, Miss, Waterways Experiment Station, 145.

who has actually read the paper

?

Comparison of correlations



SCHMERTMANN, J. H. (1975) State of the Art Paper: Measure of In situ Strength. *ASCE Conference on In situ Measurements of Soil Properties*, 2, Raleigh, North Carolina, 57-138.

SCHMERTMANN, J. H. (1978) Guidelines for Cone Penetration Test, Performance and Design, Report FHWA-TS-78-209. Washington DC, Federal Highway Administration, 145.

VILLET, W. C. B. & MITCHELL, J. K. (1981) Cone Resistance, Relative Density and Friction Angle. *Symposium on Cone Penetration Testing and Experience, ASCE National Convention*, St Louis, October, 178-208.

σ'v (kPa)	q_c Villet-Mitchell /Schmertmann (1978)			
	20%	40%	60%	80%
100	1.7	1.9	1.5	1.2
200	2.0	2.1	1.7	1.3
300	2.4	2.2	1.8	1.3

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Most famous correlation: Baldi et al. (1986)

$$D_d = \frac{1}{C_2} \ln \left[\frac{q_c}{C_o \sigma'^{C_1}} \right]$$

- Tests carried out
  - in calibration chambers
  - on Ticino and Hukksund sands
  - on normally consolidated and over consolidated sands

σ'= effective vertical stress if the sand is normally consolidated or as the effective horizontal stress or effective mean stress if the soil is over consolidated

BALDI, G., BELLOTTI, V. N., GHIONNA, N., JAMIOLKOWSKI, M. & PASQUALINI, E. (1986) Interpretation of CPT's and CPTU's - 2nd Part: Drained Penetration of Sands. *4th International Geotechnical Seminar Field Instrumentation and In-Situ Measurements*, Nanyang Technological Institute, Singapore, 25-27 November 1986, 143- 156.

Most famous correlation: Baldi et al. (1986)

Normally consolidated Ticino sand

$$D_d = \frac{1}{2.41} \ln \left[\frac{q_c}{157 {\sigma'_v}^{0.55}} \right]$$

Normally consolidated Hukksund sand

$$D_d = \frac{1}{3.29} \ln \left[\frac{q_c}{86 {\sigma'_v}^{0.53}} \right]$$

Specimens		No. of Tests	σ'	Ranges			C_o	C_1	C_2	R
				σ' (KPa)	D_R	OCR				
TICINO SAND	NC	102	σ'_v	41 to 716	0.16 to 0.96	-	157	0.55	2.41	0.96
TICINO SAND	OC	66	σ'_h	36 to 236	0.41 to 0.96	1.9 to 14.7	140	0.61	2.71	0.89
TICINO SAND	OC	66	σ'_o	46 to 260	0.41 to 0.96	1.9 to 14.7	91	0.68	2.81	0.89
TICINO SAND	NC+OC	168	σ'_h	17 to 329	0.16 to 0.96	1.0 to 14.7	220	0.53	2.64	0.94
TICINO SAND	NC+OC	168	σ'_o	26 to 458	0.16 to 0.96	1.0 to 14.7	181	0.55	2.61	0.95
HOKKSUND SAND	NC	20	σ'_v	59 to 402	0.31 to 0.98	-	86	0.53	3.29	0.98
HOKKSUND SAND	OC	15	σ'_h	27 to 97	0.26 to 0.97	2.0 to 14.5	431	0.40	2.49	0.95
HOKKSUND SAND	OC	15	σ'_o	38 to 124	0.26 to 0.97	2.0 to 14.5	204	0.58	2.44	0.96
HOKKSUND SAND	NC+OC	35	σ'_h	20 to 158	0.26 to 0.98	1.0 to 14.5	170	0.54	3.01	0.96
HOKKSUND SAND	NC+OC	35	σ'_o	33 to 217	0.26 to 0.98	1.0 to 14.5	153	0.55	2.88	0.96

Note: only tests with standard cone diameter (d_c=3.57 cm) are considered

Jamiolkowski et al. (2001)

- Ticino sand, Hukksund sand and Toyoura sand

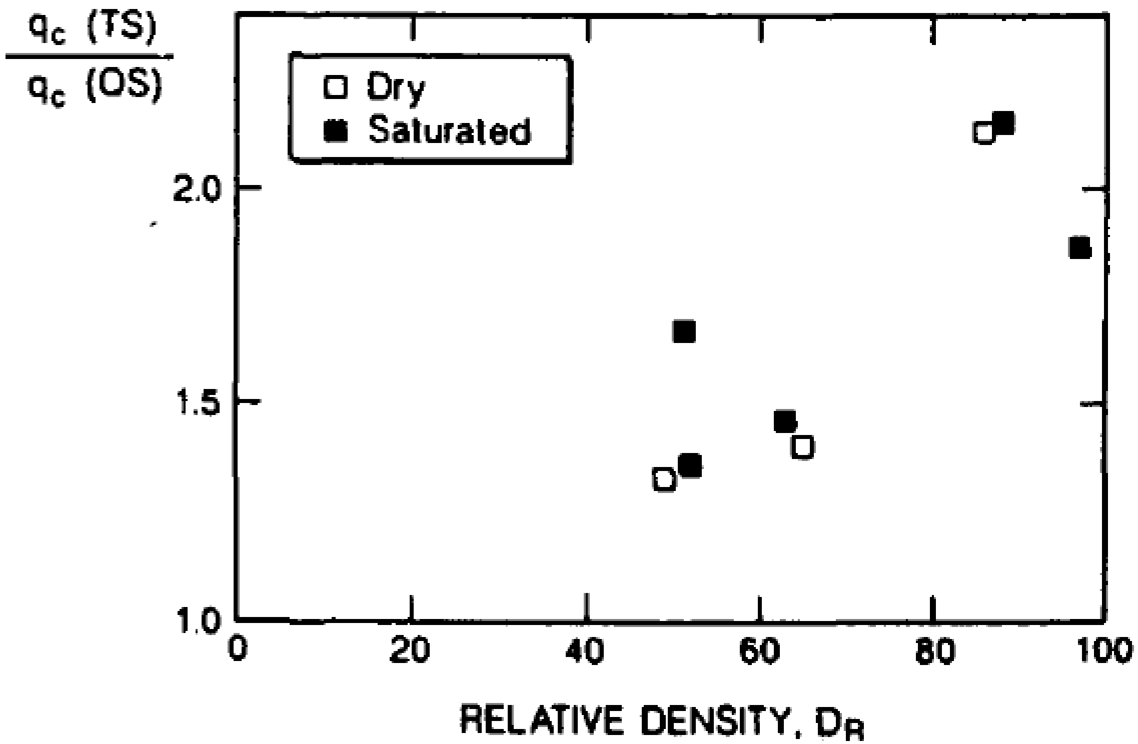
$$D_d = \frac{1}{3.10}\ln\left[\frac{q_c/98.1}{17.68\left(\sigma'_v/98.1\right)^{0.50}}\right]$$

Jamiolkowski & Pasqualini: "a quality control program based only on the evaluation of relative density can be inadequate; a better estimation of the densification of sands is possible if the effects of the stress and strain history induced on the improved soil by compaction are considered."

JAMIOLKOWSKI, M., LO PRESTI, D. C. F. & MANASSERO, M. (2001) Evaluation of Relative Density and Shear Strength of Sands from CPT and DMT. *Soil Behavior and Soft Ground Construction: Geotechnical Special Publication No. 119*, 201-238.

JAMIOLKOWSKI, M. & PASQUALINI, E. (1992) Compaction of Granular Soils - Remarks on Quality Control. *Grouting, Soil Improvement and Geosynthetics: ASCE Geotechnical Special Publication No. 30*, 2, New Orleans, 25-28 February, 902-914.

Effect of carbonate sand



Comparison of q_c of calcareous QS and silica TS at equal D_d

ALMEIDA, M. S. S., JAMIOLKOWSKI, M. & PETERSON, R. W. (1992) Preliminary Result of CPT Tests in Calcareous Quiou Sand. *Calibration Chamber Testing: First International Symposium on Calibration Chamber Testing (ISCCT1)*, Potsdam, NY, 28-29 June 1991, 41-53.

Conclusion: Do not rely on relative density as a criterion

- Due to its formulation relative density is prone to large errors
- The relationships between relative density and field tests are not unique and are strongly influenced by other parameters such as:
  - fines content
  - grain size & grain shape
  - grading & grading curve shape
  - effective vertical or horizontal stresses
  - mineralogy
  - compressibility & crushability
  - cementation, over consolidation & age

Thank You